ISI – Bangalore Center – B Math - Physics II – Back paper Exam
Date: 29 December 2015. Duration of Exam: 3 hours
Total marks: 50

Answer any 5 questions.

Q1. [Total Marks: 2+2+6 = 10]

a.) State the Kelvin-Planck and Clausius statements of the second law of thermodynamics.

b.) Show that violation of Clausius statement leads to violation of Kelvin-Planck statement.

Q2. [Total Marks: 3+3+4 - 10]

a.)From the definition of Helmholtz Free Energy F = U-TS, show that for a P,V,T system

$P = -\left(\frac{\partial F}{\partial V}\right)_T$, and $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

b.) Prove that $TdS = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV$

c.) Show that for a van der Walls gas (with equation of state $P = \frac{RT}{v-b} - \frac{a}{v^2}$) undergoing a reversible isothermal expansion from initial molar volume v_i to final molar volume v_f, the required amount of heat is given by $Q = RT\ln\left(\frac{v_f - b}{v_i - b}\right)$

Q3. [Total Marks 6+4 = 10]

a.) Using a Carnot Engine operating between isothermals T and $T + dT$ of a vapor-liquid system, derive the Clausius-Clapeyron equation

$\frac{dP}{dT} = \frac{L}{T(v_g - v_l)}$, where L is the latent heat of liquid to gas transition, and v_g , v_l are the molar volume for vapor and liquid respectively.

b.) By applying the above equation in part a.) to a liquid-solid system, describe how this equation connects the decrease in melting temperature of ice under pressure with the anomalous expansion of water.

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Q4. [Total Marks 2+2+3+ 3= 10]

a.) Define Boltzman Partition function Z for a thermodynamic system.

b.) Derive the expression for average energy of a system in equilibrium at temperature T in terms of derivatives of the partition function

c.) Consider a model of paramagnetic solids in a magnetic field B in which there are particles with spin located at N sites and at each site only two states are possible, spin up (with energy $-\mu B$) and spin down (with energy $+\mu B$). Show that the average energy for this system is given by

$$-N\mu B \tanh\left(\frac{\mu B}{kT}\right)$$

d.) Derive the expression for specific heat for this system. What is the limiting value of the specific heat as T goes to zero?

Q5. [Total Marks 2+4+4=10]

a.) Mention two experiments that show that on reflection from optically rarer to optically denser medium, light undergoes a phase transition of π.

b.) Assume that there is no such phase transition during reflection from optically denser to rarer medium or during refraction. Let *a* be the amplitude of an incident ray and $-ar$ and *at* be the amplitudes of reflected and refracted rays when light goes from a optically rarer to an optically dense medium. Similarly let ar be $a\bar{t}$ at be the corresponding amplitudes when light goes from the denser to rarer medium (same pair of medium).

Show using principle of reversibility that

$$t\bar{t} = 1 - r^2$$

c.) Using part b.) above show that for a thin film with thickness $\frac{\lambda}{2n}$, there will be complete destructive interference at normal incidence when all the reflected rays are taken together.

Q6. [Total Marks 5+5=10]

a.) Young's double slit experiment is performed with orange light with wavelength 6057.8 Angstrom (1 Angstrom $=10^{-8}$ cm). It is found that on a screen 100 cm away from the double slits, 25 fringes occupy a distance of 12.87 mm. How are apart are the two slits?

b.) When Young's double slit experiment is conducted with coherent white light , it is found that there is a bright fringe followed very closely by less bright red fringe followed by a violet fringe and after which there is no discernible fringes seen. Explain this phenomenon.